

separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.

Animals such as chickens have easier systems to control. [Figure 9.17](#) shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

[Figure 9.17](#) shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

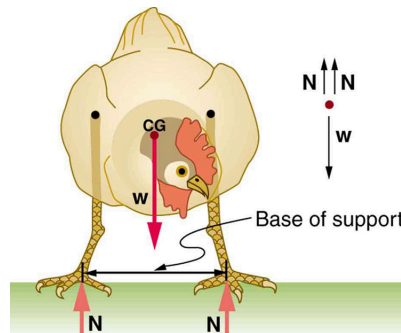


Figure 9.17 The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

Take-Home Experiment

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

9.4 Applications of Statics, Including Problem-Solving Strategies

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in [Problem-Solving Strategies](#), still apply.

Problem-Solving Strategy: Static Equilibrium Situations

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.

2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations $\text{net } \mathbf{F} = 0$ and $\text{net } \boldsymbol{\tau} = 0$, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$)). Always choose a convenient coordinate system for projecting forces.
4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In [Figure 9.18](#), the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium ($\text{net } \mathbf{F} = 0$). The second condition ($\text{net } \boldsymbol{\tau} = 0$) is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In [Figure 9.18](#), a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See [Figure 9.18](#). If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

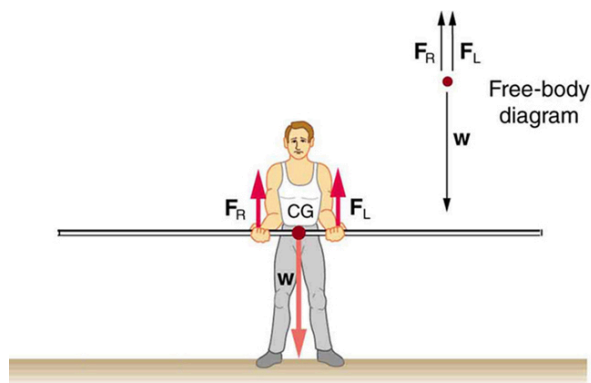


Figure 9.18 A pole vaulter holds a pole horizontally with both hands.

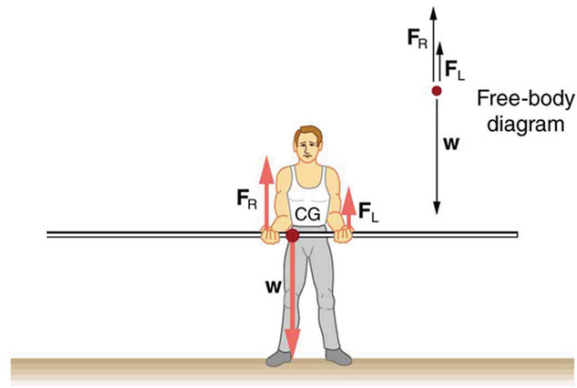


Figure 9.19 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.

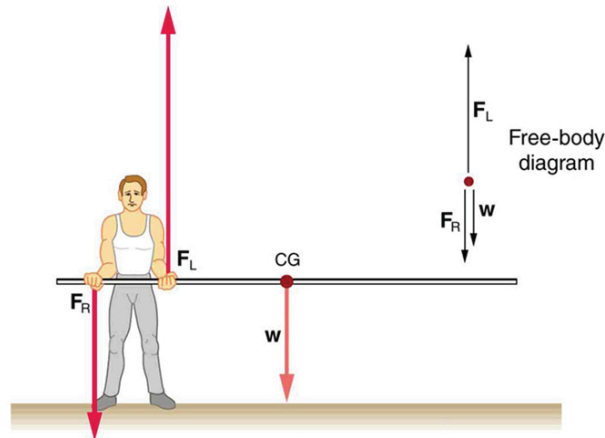


Figure 9.20 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in [Figure 9.19](#), the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $F_L = F_R$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces F_L and F_R is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole ([Figure 9.20](#)), the direction of the force applied by the right hand of the vaulter reverses its direction.



EXAMPLE 9.2

What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in [Figure 9.19](#), calculate: (a) F_R , the force exerted by the right hand, and (b) F_L , the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

[Figure 9.19](#) includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium (net $\mathbf{F} = 0$), since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium (net $\tau = 0$) if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force (τ_w) and from the push or pull of the right hand (τ_R). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\text{net } \tau_{cw} = -\text{net } \tau_{ccw}.$$
9.22

or the algebraic sum of the torques is zero.

Here this is

$$\tau_R = -\tau_w$$
9.23

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque, $\tau = rF \sin \theta$, noting that $\theta = 90^\circ$, and substituting known values, we obtain

$$(0.900 \text{ m})(F_R) = (0.600 \text{ m})(mg).$$
9.24

Thus,

$$\begin{aligned} F_R &= (0.667)(5.00 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 32.7 \text{ N}. \end{aligned}$$
9.25

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$F_L + F_R - mg = 0$$
9.26

From this we can conclude:

$$F_L + F_R = w = mg$$
9.27

Solving for F_L , we obtain

$$\begin{aligned} F_L &= mg - F_R \\ &= mg - 32.7 \text{ N} \\ &= (5.00 \text{ kg})(9.80 \text{ m/s}^2) - 32.7 \text{ N} \\ &= 16.3 \text{ N} \end{aligned}$$
9.28

Discussion

F_L is seen to be exactly half of F_R , as we might have guessed, since F_L is applied twice as far from the cg as F_R .

If the pole vaulter holds the pole as he might at the start of a run, shown in [Figure 9.20](#), the forces change again. Both are considerably greater, and one force reverses direction.

Take-Home Experiment

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

**PHET EXPLORATIONS****Balancing Act**

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

[Click to view content \(https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html\)](https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html)

Figure 9.21



9.5 Simple Machines

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for “machine” comes from the Greek word meaning “to help make things easier.” Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$MA = \frac{F_o}{F_i}$$

9.29

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

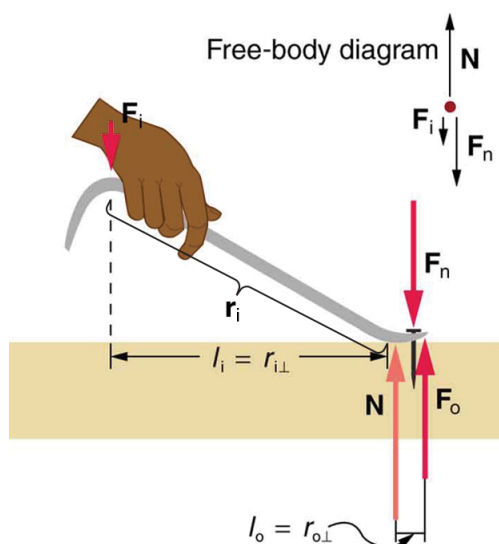


Figure 9.22 A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail (F_o) is not a force on the nail puller. The reaction force the nail exerts back on the puller (F_n) is an external force and is equal and opposite to F_o . The perpendicular lever arms of the input and output forces are l_i and l_o .

Figure 9.22 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. F_i is the input force and F_o is the output force. There are three vertical forces acting on the nail puller (the system of interest) – these are F_i , F_n , and N . F_n is the reaction force back on the system, equal and opposite to F_o . (Note that F_o is not a force on the system.) N is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to F_i and F_n must be equal to each other if the nail is not moving, to satisfy the second condition for equilibrium (net $\tau = 0$). (In order for the nail to actually move, the torque due to F_i must be ever-so-slightly greater than torque due to F_n .) Hence,

$$l_i F_i = l_o F_o$$

9.30

Notice that r_i is the distance from the pivot point to the point where the input force F_i is applied, and r_o (not labeled on the diagram) is the distance from the pivot point to the point where the output force F_o is applied. The distances l_i and l_o are the perpendicular components of the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_o}{F_i} = \frac{l_i}{l_o}$$

9.31